UPKAR'S
CSIR-UGC
NET/JRF/SET
MATHEMATICAL
SCIENCES

By

Dr. Alok Kumar
Institute of Engineering & Technology,
Dr. B. R. Ambedkar University, Agra

Latest Revised Edition

A WORD TO THE READER

The National Eligibility Test (NET) for Junior Research Fellowship and Eligibility for Lectureship are conducted by National Testing Agency (NTA) twice a year is one of the most competitive examinations in India. Different states also now have started conducting this examination for their own examinations under State Eligibility Test (SET).

About the Examination

The NET will have a single paper with Multiple Choice Questions, divided in three parts :

Part-A : Marks 30

Part-B : Marks 75

Part-C : Marks 95

Part-A : This part shall carry 20 questions pertaining to General Science, Quantitative Reasoning & Analysis and Research Aptitude. The candidates shall be required to answer any 15 questions. Each question shall be of two marks. The total marks allocated to this section shall be 30 out of 200.

Part-B : It will have 40 objective type questions from the subject area. The candidate of Mathematical Sciences will have to attempt any 25 questions out of the given 40 questions, each carry 3 marks. **Negative marking will be made for wrong answers.**

Part-C : It will consist of a number of questions with multiple correct options. Credit in a question shall be given only on identification of all correct options. (Confined to a maximum of one page length). The candidate will be required to answer 20 questions. Each question is of 4·75 marks. The total section is of 95 marks.

The level expected at the examination is that of post-graduate level.

The specified number of questions are only evaluated.

How will this book help you ?

Primarily meant for CSIR-UGC NET candidates of Mathematical Sciences as per revised syllabus, this edition thoroughly covers all the topics essential for students of MATHEMATICS, OPERATIONS RESEARCH and STATISTICS stream, as prescribed in the CSIR-UGC : NET Syllabus.

Consistence is maintained and each chapter is kept independent to each other, candidate are required to go through the inconsistences too since subject matter differs as per need from chapter to chapter.

We are sure that candidate appearing for the examinations will find this book extremely useful in preparation of paper and achieving their desired goal.

At last author wish to acknowledge Mr. Mahendra Jain, Publisher UPKAR PRAKASHAN. The author is also grateful to Dr. Madhu Jain and Professor G. C. Sharma for their valuable suggestions and discussion on the subject. The author also gratefully acknowledge and express deep appreciation to the many wonderful authors of Mathematical Sciences without consulting their valuable books it is impossible to write this book. Any suggestions for further improvement and queries regarding this book is highly welcomed.

With best wishes,

—Dr. Alok Kumar

Contents

(v)

GENERAL INFORMATION

SCHEME OF EXAMINATION

Time : 3 Hrs.   **Max. Marks :** 200

Single Paper Test having Multiple Choice Questions (MCQs) is divided in three parts.

Part 'A'

This part shall carry 20 questions pertaining to General Science, Quantitative Reasoning & Analysis and Research Aptitude. The candidates shall be required to answer any 15 questions. Each question shall be of two marks. The total marks allocated to this section shall be 30 out of 200.

Part 'B'

This part shall contain 40 Multiple Choice Questions (MCQs) generally covering the topics given in the syllabus. A candidate shall be required to answer any 25 questions. Each question shall be of three marks. The total marks allocated to this section shall be 75 out of 200.

Part 'C'

This part shall contain 60 questions that are designed to test a candidate's knowledge of scientific concepts and/or application of the scientific concepts. The questions shall be of analytical nature where a candidate is expected to apply the scientific knowledge to arrive at the solution to the given scientific problem. The questions in this part shall have multiple correct options. Credit in a question shall be given only on identification of ALL the correct options. No credit shall be allowed in a question if any incorrect option is marked as correct answer. No partial credit is allowed. A candidate shall be required to answer any 20 questions. Each question shall be of 4.75 marks. The total marks allocated to this section shall be 95 out of 200.

● For Part 'A' and 'B' there will be Negative marking @25% for each wrong answer. No Negative marking for Part 'C'.

● Model Question Paper is available on HRDG website www.csirhrdg.res.in.

SYLLABUS
PART–A

This part shall carry 20 questions pertaining to General Science, Quantitative Reasoning & Analysis and Research Aptitude. The candidates shall be required to answer any 15 questions. Each question shall be of two marks. The total marks allocated to this section shall be 30 out of 200.

(Common Syllabus for Part 'B and C')
UNIT – I

Analysis : Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum.

Sequences and series, convergence, limsup, liminf.

Bolzano Weierstrass theorem, Heine Borel theorem.

Continuity, uniform continuity, differentiability, mean value theorem.

Sequences and series of functions, uniform convergence.

Riemann sums and Riemann integral, Improper Integrals.

Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral.

Functions of several variables, directional derivative, partial derivative, derivative as a linear

transformation, inverse and implicit function theorems.

Metric spaces, compactness, connectedness. Normed Linear Spaces. Spaces of Continuous functions as examples.

Linear Algebra : Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations.

Algebra of matrices, rank and determinant of matrices, linear equations.

Eigen values and eigen vectors, Cayley-Hamilton theorem.

Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms.

Inner product spaces, orthonormal basis.

Quadratic forms, reduction and classification of quadratic forms.

UNIT – II

Complex Analysis : Algebra of complex numbers, the complex plane, polynomials, Power series, transcendental functions such as exponential, trigonometric and hyperbolic functions.

Analytic functions, Cauchy-Riemann equations.

Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem.

Taylor series, Laurent series, calculus of residues.

Conformal mappings, Mobius transformations.

Algebra : Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements.

Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler's ϕ- function, primitive roots.

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems.

Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain.

Polynomial rings and irreducibility criteria.

Fields, finite fields, field extensions, Galois Theory.

Topology : Basis, dense sets, subspace and product topology, separation axioms, connectedness and compactness.

UNIT – III

Ordinary Differential Equations (ODEs) : Existence and Uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs.

General theory of homogeneous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Partial Differential Equations (PDEs) : Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs.

Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Numerical Analysis : Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Calculus of Variations : Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

Linear Integral Equations : Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigen functions, resolvent kernel.

Classical Mechanics : Generalized coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action, Two-dimensional motion of rigid

bodies, Euler's dynamical equations for the motion of a rigid body about an axis, theory of small oscillations.

UNIT – IV

Descriptive Statistics, Exploratory Data Analysis : Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (i.i.d. case).

Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson and birth-and-death processes.

Standard discrete and continuous univariate distributions. Sampling distributions. Standard errors and asymptotic distributions, distribution of order statistics and range.

Methods of estimation. Properties of estimators. Confidence intervals. Tests of hypotheses : most powerful and uniformly most powerful tests, Likelihood ratio tests. Analysis of discrete data and chi-square test of goodness of fit. Large sample tests.

Simple non-parametric tests for one and two sample problems, rank correlation and test for independence. Elementary Bayesian inference.

Gauss-Markov models, estimability of parameters, Best linear unbiased estimators, confidence intervals tests for linear hypotheses. Analysis of variance and covariance. Fixed, random and mixed effects models. Simple and multiple linear regression. Elementary regression diagnostics. Logistic regression.

Multivariate normal distribution, Wishart distribution and their properties. Distribution of quadratic forms. Inference for parameters, partial and multiple correlation coefficients and related tests. Data reduction techniques : Principle component analysis, Discriminant analysis, Cluster analysis, Canonical correlation.

Simple random sampling, stratified sampling and systematic sampling. Probability proportional to size sampling. Ratio and regression methods.

Completely randomized designs, randomized block designs and Latin-square designs. Connectedness and orthogonality of block designs, BIBD. 2k factorial experiments : confounding and construction.

Hazard function and failure rates, censoring and life testing, Series and parallel systems.

Linear programming problem. Simplex methods, duality. Elementary queuing and inventory models. Steady-state solutions of Markovian queuing models : M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1.

All students are expected to answer questions from Unit-1. Mathematics students are expected to answer additional questions from Unit-II and III. Statistics students are expected to answer additional questions from Unit-IV.